Computer Understanding of Venn and Euler Diagrams

Abstract—Venn & Euler diagrams are well-defined mathematical diagram types, which are the major representation methods of Set Theory. Although understanding of different diagram types such as charts and coordinates graphs has been addressed, no research has been done for Venn and Euler diagram interpretation from an image. Venn and Euler Diagrams exist in various media types such as printed format in books, raster images and vector images in electronic media. In this research, interpretation method is applied to images in vector format. Methodology for Set details extraction from a vector image is presented and Venn Data representation is introduced, which can store Venn details extracted from a Venn or Euler diagram.

Keywords— Diagram Understanding, Venn Diagram, Euler Diagram, Set Theory, Vector Images

1. Introduction

Diagrams are a very important communication medium. This is especially the case with Mathematical diagrams. Although humans can easily interpret these diagrams, mathematical diagram understanding is a complex challenge for researchers. Diagram understanding is an important pre-requisite of various fields such as image database systems, and educational diagram grading systems. Significant research has been done to understand mathematical diagrams in few domains such as coordinate graphs [2, 5] as well as charts (bar chats, pie charts). However there is no significant research done to interpret Venn and Euler diagrams.

We address the problem of computer understanding of Venn and Euler diagrams that are available in vector format. Venn Diagrams have been developed by John Venn (1843 - 1923) and became a common representation tool in proportional logic and related branches of mathematics such as Boolean Algebra [7]. Venn diagrams are a specialized instance of a more general notation for representing relationships among a set of classes of concepts referred to as Euler diagrams developed by Leonard Euler.

Definition of Venn and Euler diagrams varies throughout the literature [20]. Generally, Venn and Euler diagrams can be defined as a finite set of labelled, closed curves. The closed curves in the diagram partition the plane into minimal regions, where each minimal region is a connected component of the plane inside a set of curves [20]. While a Venn diagram contains all the minimal regions, Euler diagrams omit the empty minimal regions. Normally 2 and 3 sets of Venn and Euler diagrams are being used in most of educational context. Representation up to 6 sets is less complex in the graphical representation. Fig. 1 Shows a Venn diagram and the relevant Euler diagram of 2 sets for which the set of A∩B is empty.



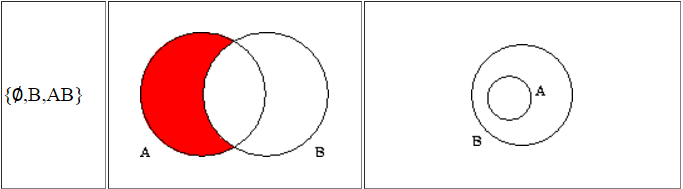


Fig. 1 Example of a Venn and an Euler Diagram

To address the problem of computer understanding of Venn and Euler diagrams, we present a methodology to extract set details from a vector image and produce the output as an xml structure. Our system accepts the input images as vector images. Under this representation, polygons are used to represent an image [21]. Vector graphics are defined on x-y axes on a plane. Vector graphics are the main media of information graphics. Since the W3C standard of vector graphics is SVG, we accepted the input as SVG images. SVG has a XML structure, capable of supporting any Venn diagram representation and supports all major browsers, and mobile devices, and is scalable.

In vector images, sets are identified by labelled Jordan Curves that are non-self-intersecting continuous closed paths in the plane [20]. Normally, in SVG sets can be represented with circles, ellipses, rectangles and closed curves. In this research we only consider circles, ellipses and rectangles. However, same concepts can be extended for the arbitrary closed curves as well, though it is not currently handled. A set sometimes has an attached label. In the label identification, only nominal text labels are accepted as labels. Since the label identification is subjected to various ambiguity problems, various heuristics had to be used to identify correct labels.

After identifying the sets, minimal regions are identified. Then remaining text is classified as zone elements. After extracting all the Venn information, extracted data is output in a structured XML format that can describe any Venn diagram. It contains set details and the zone details. Arrangement difference in the vector image of the Venn diagram can produce several XML outputs that are equivalent.

This method has been tested against the Venn and Euler diagrams produced from university students and secondary school students. We gave Venn and Euler diagram question paper with four questions taken from the Mathematics paper from the GCE O/L examination in Sri Lanka to university undergraduate and postgraduate students and collected answers in hand written format. Also we have collected answer from a secondary school examination that has Venn and Euler diagram questions. Collected answer sheets were converted to electronic format (SVG) before parsing. The diagrams were parsed using the system. Parsed output was manually checked for each diagram for the validation of the system, which showed an accuracy of 89.61%.

This paper is arranged into following sections: Section II describes the related work. Abstract solution details of the complete research, the proposed solution and how the information extraction methods were applied are included in Section III. Section IV provides an evaluation of the system. Finally, Section V concludes the paper with possible future extensions.

1. Related Work

In this section, we explore existing research related to mathematical diagram recognition, interpretation, diagram similarity measurements and diagram data representation methods.

Mathematical diagram recognition, understanding and evaluation is a relatively new research field. As an early attempt, Futrelle et al [1] presented a diagram understanding system to interpret diagrams based on constraint grammars. The system is capable of handling x-y graphs and gene diagrams on Biological domain [2]. Tsintsifas et al [8] developed a java based framework called DATsys that can be used to understand and evaluate diagrams. They were able to develop the diagram input system that can be scalable to various diagram types. This system was developed as an extension to existing Ceilidh Computer Assessment System [17]. Thomas et al [9, 10, and 12] developed a computer aided assessment system that can handle graph based diagrams such as Entity-Relationship diagrams, flow charts where information can be represented as data nodes and relations between. Diagrams are interpreted using basic set of units called “Minimal Meaningful Units” and introduced a marking criteria based on “Minimal Meaningful Units” which can be applied for diagrams that can be represented using nodes and their relationships. Tsintsifas et al [8] developed a method to assess diagrams in formative assessment in which primary aim was to assist the process of learning. They followed the work of Brett et al [15] and developed a feedback system that can work with graph based diagrams. They also discussed a simple evaluation method and developed a grammar for E-R diagrams. Batmaz et al [18] developed a diagram drawing tool that can be used for semi-automated database diagram assessment. Huang et al [4, 5] developed a system that can understand chart images. They were able to recognize and identify various types of chart images (both 2D and 3D) and interpret those images and produce an xml output which can be used for further processing. Research from Futrelle et al [1] and Huang et al [4] worked with raw pixel images that were extracted from hard documents. They converted those pixel images to Vector Graphics format for further processing.

In the area of diagrammatic reasoning Anderson et al [16] discussed the fundamental components of a diagrammatic processing system, (1) Means to input diagrams that can be a vision component or direct link to a diagram source, (2) Diagram representation to internally represent diagrams, (3) Storage management component and (4) Processing component that synthesizes and abstracts new knowledge from combinations of diagrammatic and other forms of knowledge representations. In early research, they dealt with many types of input system types based on their research focus.

When it comes to vector based input systems, text label processing is a tricky problem. Some diagram based assessment systems have added restrictions to the label choice in order to minimize the association problem. Diagram drawing tool by Batmaz et al [18] provides a set of labels that can be selected as inputs. In DATsys [8] and OpenMark [9] they provide label insertion areas but the user can decide the laberl he want to enter. Since labelling has more freedom, text association is a hard problem. Futrelle et al [2] discussed the object association problem and heuristic approach development using special techniques in general. Huang et al [5] developed a machine learning technique based on the decision graph technique to pre-process texts to help text association. Also in the diagram marking text similarity measure is also important. Jayal et al [13] discussed the label similarity problem from Natural Language Processing aspect.

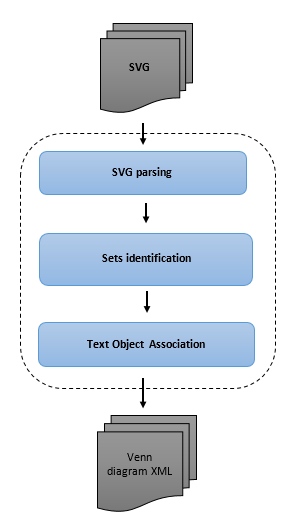
Fig. 1 SVG XML structure of a Venn diagram

None of above research address the issue of Venn or Euler diagram interpretation and we are not aware of any other general Venn or Euler diagram interpretation research work.

1. Venn and Euler Diagram Parser

In this section we describe the implementation details of the Venn and Euler diagram parser.

1. High Level Architecture

ig. 2 shows high level architecture of the implemented system. Input diagram is given in the SVG format. It contains geometric details of SVG primitive objects such as circle, ellipse and rectangle, as well as other details such as presentation details. Since it contains presentation and SVG specific data, SVG image has to be parsed and primitive shape details should be extracted. These details are needed to build Venn and Euler data information.

Parsed SVG contains mostly geometrical details such as size, primitive object geometry details and text label details. F From the parsed SVG, sets are identified by finding the Jordan curves. In this case, only circles, rectangles and ellipses are considered. After finding the sets, relevant set labels are identified using a heuristic algorithm (More details can be found at the “Set Label Identification” section). Only nominal labels are considered as valid set labels. Since text labels can be categorized into few types such as title, set labels and set elements, few heuristic parameters are considered to classify the set labels using some parameters such as closeness to the set area boundary, nominal/ numeric labels, font size and relative position to set areas. If sets are labelled using arrows, relevant arrows are identified and the associated text with the arrow is considered as the set label. Generated temporary label is given to the set in case of there is no associated set label.

After identifying the sets, all the minimal regions are identified and minimal region geometric properties such as centroid of the minimal region area and size of the area are calculated for text association building.

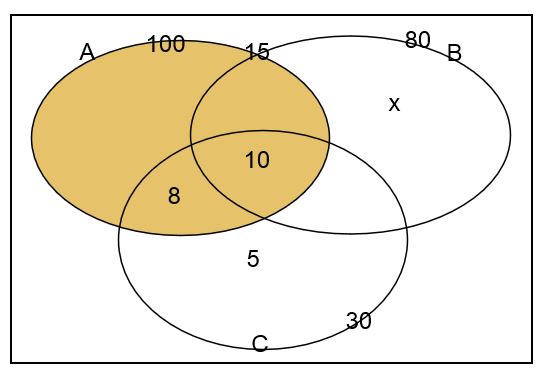
Then the text labels are associated with the correct minimal regions or zones (set of minimal regions) based on the Venn diagram domain knowledge. Several heuristic algorithms are used to deal with human errors. More details on this can be found at the “Text Association Mapping” section. Heuristic parameter tuning depends on the source of the Venn and Euler diagrams such as distance closeness parameters depends on the font sizes based on the SVG drawing tool.

Fig. 3 Venn diagrams as an SVG image

After associating the text labels, Venn diagram is built using the extracted knowledge and output is created as structured XML which is able to contain only the Venn and Euler details without the initial Venn diagram presentation details such as orientations, and set curve shapes.

1. SVG Parsing

SVG format is the standard W3C standard of vector graphics. SVG format is capable of handling Venn diagrams. (Fig. 3 shows a Venn diagram drawn in SVG format).

Fig. 2 High Level Architecture of the tool

Fig. 4 shows the XML structure of the Venn diagram presented in Fig 3. SVG of a Venn diagram contains basic mandatory SVG features and optional details such as size and file type, Grouping details, image titles, primitive object (rectangle, line, circle …etc.) shape details, primitive object presentation details (fill, stroke details …etc.) and text label details. In SVG parsing only details required to generate Venn information are retained.

1. Set identification

In Venn and Euler diagrams, sets are represented with Jordan curves (non-self-intersecting closed curves). In SVG diagrams, there are several possible primitive objects such as circles, rectangles, ellipses and closed paths that can be considered as Jordan curves. We identified those closed curves in the SVG diagram from the SVG objects. In this research, only the circles, rectangles and the ellipses are considered since majority of the Venn and Euler diagrams are drawn using those shapes [20]. However, the same methodology can be extended to other primitive objects that act as Jordan curves.

1. Set Label Identification

After identifying sets, associated set labels have to be identified. Sets can be labelled in two ways; with arrows and without arrows by putting the text label near the boundary of a set area. If labelled with arrows, arrow ending has to exist near the set area boundary. If associated arrow is found for any set, then the associated text with the arrow tail is considered as the set label.

If there is no associated arrow for a set, then the closest nominal label near the boundary of the set area is considered as the set label. Fig 6. shows the boundary condition check for an ellipse. In the ellipse boundary condition check, only the centre (C) coordinates and horizontal radius (a) and vertical radius (b) are given in the SVG. Using these details, focal points (F1, F2) and the sum of distance to any point on the ellipse from focal points are calculated (Eq. 1, 2 and 3) from mathematical properties of an ellipse (Fig. 5 shows the required mathematical properties).

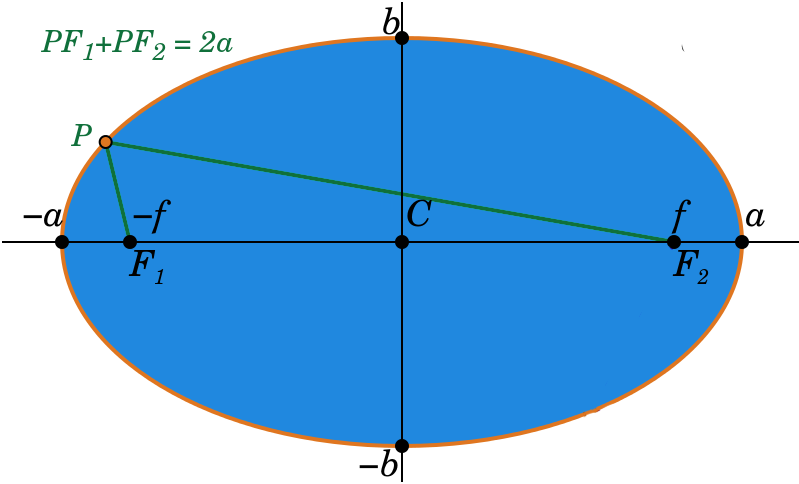
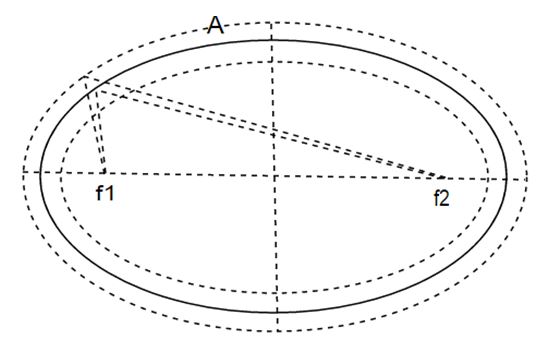


Fig. 5 Mathematical properties of an ellipse.

--- Eq. (1)

--- Eq. (2)

Fig. 6 Boundary condition check for an ellipse.

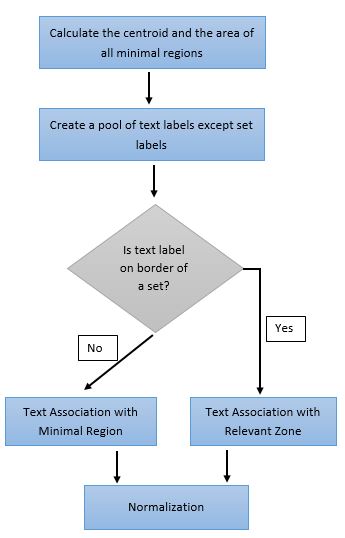


--- Eq. (3)

These are used to check the boundary conditions. The text labels appearing in the narrow area near a set boundary are considered as possible set labels for the considered set. From text labels that exist only in the considered set area boundary, the closest nominal label is selected as the label for the corresponding set. Sets that do not have a text label are given a computer generated label for further processing.

1. Text Association Mapping

In a Venn or Euler diagram, elements associated with minimal regions are marked on the minimal region area. Since the minimal regions are constructed from combinations of set boundary parts, minimal regions normally have complex boundaries. Since identification of minimal region boundaries is a complex task, minimal region centroid and the area of the minimal region is approximated by counting coordinates belonging to each region.



Then, the possible text that can be an element of a minimal region is filtered using the centroid and minimal region area. From the filtered text elements, correct text elements that are in the region are identified.

Fig. 7 Text association mapping

1. Venn Data Structure

Fig. 8 shows the XML schema of the Venn and Euler data output. In theVenn data XML structure, there are five top level tags; (1) type: Type of the diagram (Ex: - “Venn Diagram”), (2) Title: Diagram Title Label (If any), (3) no\_of\_sets: Number of sets in the diagram, (4) sets: set of sets, (5) data\_set: minimal region and zone data.

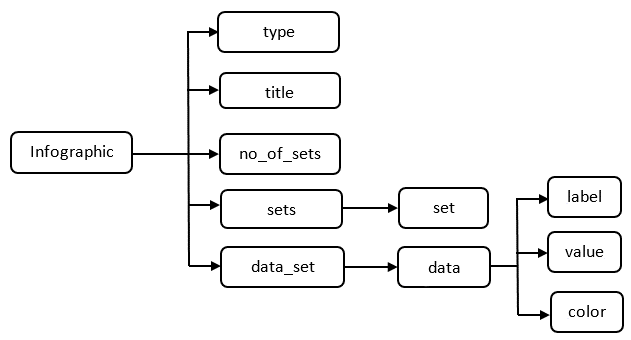
Data set order is ignored in the Venn information XML structure. Depending on the set order, different Venn or Euler diagrams may produce equivalent solutions. When reading the output XML ignoring the order of sets, it will produce the same Venn or Euler data.

Fig. 8 Venn/ Euler data output XML structure

1. Implementation and Analysis
2. *Implementation*

The parser is developed using Java. Input Venn or Euler diagram images are in SVG format. In the parser, SVG model is present with only relevant details of SVG attributes such as geometric details. Most of the styling details are omitted. Conversion to the relevant set objects from an SVG file is done in the initial phase using Java XPath.

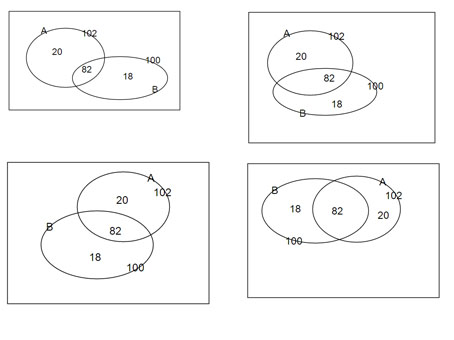


Fig. 9 Variation of Venn diagrams representing the same set information

Circles, rectangles, ellipses are identified as sets in the set identification phase from the SVG model. Minimum font size of a SVG image is extracted using all the text labels. In the set label mapping, closeness to the boundary of a set has upper limit of T1 \* MINIMAL\_FONT\_SIZE. Nominal label closest only to a set boundary is considered as the set label when arrow labelling is not present. Arrows are presented as lines in SVG. If an arrow ending exists near a boundary of a set, closest nominal label (up to the upper limit of T2 \* MINIMAL\_FONT\_SIZE) of the other end of the arrow is considered as the set label. After set identification, sets without any label are given a generated label name. In the experiment T1 boundaryparameter is selected as 1.5 and T2 boundary parameter is selected as 5 based on the tuning phase results since closeness depends on the image scale.

Fig. 10 XML representation for the Venn diagrams given in the Fig. 9

After the text label association with the relevant minimal regions and zones, object oriented model of the Venn data is built and the output is generated as an XML format.



1. *Analysis*

For a given set, output xml can have a limited number (n!) of equivalent formats depending on the arrangement of sets. Variation of Venn or Euler diagram for 2 sets produce 2 equivalent XML formats.

Since in the set theory set order details are not significant, by omitting the sets order in the xml Venn data, equivalency of two diagrams can be measured. Fig 9 shows few variations to the same set information. Fig 10 shows XML representation for the Venn diagrams given in the Fig. 9.

Venn parser is tested and tuned using the Venn diagrams gathered from university undergraduates, which are drawn directly using an SVG editor. For the evaluation, we collected hand written answer scripts which collectively contained 3 Venn and 4 Euler diagrams from university undergraduates and grade 10 school students. In total, we hads 77 Venn & Euler diagrams. Those diagrams were hand drawn. Therefore they were converted to Vector format using an SVG editorbefore given to the parser. Parser successfully parsed 69 of those diagrams correctly having a collective accuracy of 89.61%.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Diagram No. | Diagram Type | Correctly Parsed | Incorrectly Parsed | Accuracy |
| 1 | Venn | 17 | 2 | 89.5% |
| 2 | Venn | 12 | 0 | 100% |
| 3 | Euler | 11 | 0 | 100% |
| 4 | Euler | 11 | 0 | 100% |
| 5 | Euler | 3 | 2 | 60.0% |
| 6 | Euler | 5 | 3 | 62.5% |
| 7 | Venn | 10 | 1 | 90% |

Diagram 1 and 2 contain 2 sets and remaining diagrams contains 3 sets. Diagram 1 collected from school student answer scripts and remaining from university undergraduates. Diagram 5 and 6 had fewer answers because of the higher complexity of the question. All of the parsing errors are due to the ambiguity of the text labels and some text labels being too far away from the arrows.

1. Conclusions
2. Future Work

Number of possible further extensions can be identified with respect to the implemented system.

In this research, sets can be drawn from few shapes such as rectangles, circles and ellipses. This method can be extended to apply for sets drawn with any type of Jordan curves. In some Venn and Euler representations, one set label can have more than one Jordan curve. This method can be extended to address those representations.

Machine learning approaches can be used to improve the label classification and clustering. Currently machine learning methods are not used since we do not have significantly large vector image database.

This method can be easily extended into other vector formats of images, since conversion methods of pixel images format into vector format are already developed. Also this method can be extended to parse printed images.

1. Concluding Remarks

Diagram understanding is a complex problem in the computer research field. Structured diagrams can be dealt with using the domain ontology, but the unstructured drawing understanding is a very complex problem. In particular, dealing with human made diagrams is difficult due to the ambiguity problems and human errors that exist in those diagrams. Moreover, for some diagrams, diagram definitions are not standardized. Having various structured and unstructured notations for the same diagram types makes it more complex to address diagram understanding.

In this research we have successfully established the required methods to interpret Venn or Euler diagrams represented in SGV vector format and introduced a generic format to represent a Venn or Euler diagram. We believe that solution we have introduced will help to develop systems related to image understanding such as automatic grading systems and image database systems.

Acknowledgment

< Will be provided upon paper assessment>

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